Reference: Communication systems-Simon Haykin (2001)

Chapter2:

In chapter1, we investigated the way of modulating a sinusoidal carrier wave using AM technique.
There is another way of modulating a, sinusoidal carrier wave, namely, **angle modulation** in which the angle of the carrier wave is varied according to the base-band signal.
In this method of modulation the **amplitude** of **the carrier wave** is maintained **constant**.
An important feature of angle modulation is that it can provide better discrimination against noise and interference than amplitude modulation.
However, this improvement in performance is achieved transmission bandwidth.

Definition:

If $\theta\,(t)$ devote the angle of a modulated sinusoidal carrier, we express the resulting **angle modulation** wave as

$$\boxed{s(t) = A_C \cos(\theta_i(t))} \qquad (2.1)$$

Where AC is the carrier amplitude, $\theta(t)$ – phase of the signal. $\theta(t)$ assumed to be a function of the message signal.

A complete oscillator occurs whenever $\theta(t)$ changes by 2π radians.
We may define the instantaneous frequency of the angle- modulated signal s(t) as follows:

$$\boxed{f_i(t) = \frac{1}{2\pi}.\frac{d\theta(t)}{dt}} \qquad (2.2)$$

$$\therefore\ W_i(t) = 2\pi f_i(t) = \frac{d\theta_i(t)}{dt}$$

According to equation 2.1, we may interpret the angle modulated signal $s(t)$ as a rotating phasor of length A_C and an angle $\theta(t)$. the angular velocity of such a phasor is $\frac{d\theta(t)}{dt}\frac{rad}{\sec}$ (i.e. equation 2.2)

In the simple case of unmodulated carrier, the angle $\theta(t)$ is $\theta_i(t) = 2\pi f_C t + \phi_C$ initial phase (constant) - 2.3

and the corresponding phaser rotates with a constant angular velocity equal to $2\pi f_c$

$\therefore\ \frac{d\theta_i(t)}{dt} = 2\pi f_C$ = (constant) ,The value of $\theta_i(t)$ at t = 0 is ϕ_C.

There are an infinite number of ways in which the angle $\theta(t)$ may be varied in some manner with the message signal.

However ,we shall consider only two commonly used method, **phase modulation** and **frequency modulation** .

Phase Modulation(PM)

It is a form of angle modulation in which the angle $\theta_i(t)$ is varied **linearly** with the message signal m(t),

$$\theta_i(t) = 2\pi f_C\, t + \phi(t)$$

carrier — not a constant

Where $\phi(t) = k_P m(t)$

$$\therefore \theta_i(t) = 2\pi f_C t + k_P m(t) \qquad (2.3)$$

The term $2\pi f_C t$ represents the angle of the carrier ; the constant k_P **represents the phase sensitivity** of the modulator , expressed in radians per volt on the assumption that m(t) is a voltage waveform.

The phase modulated signal s(t) is thus described in the time domain by

$$S\ (t) = A_C \cos[2\pi f_C t + k_P m(t)] \qquad (2.4)$$

Frequency modulation(FM)

Frequency modulation is that from of angle modulation in which the instantaneous frequency $f_i\ (t)$ is varied linearly with the message signal m(t):

$$f_i(t) = f_C + k_f m(t) \qquad (2.5)$$

The term f_C represents the frequency of the unmodulated carrier and the constant k_f represents the **frequency sensitivity** of the modulator, expressed in hertz per volt on assumption that m(t) is a voltage waveform .

Integration equation (2.5)with respect to time we get,(after multiplying by2π).

$$\theta_i(t) = 2\pi f_C + 2\pi k_f \int_0^t m(\tau)d\tau \qquad (2.6)$$

The frequency modulated signal is therefore described in the time domain by

$s(t) = A_C \cos(\theta_i(t))$

$$s(t) = A_C \cos\left[2\pi f_C t + 2\pi k_f \int_0^t m(c)dc\right] \qquad (2.7)$$

*In summary (see equation 2.4 & 2.7)

Angle modulated wave

$$s(t) = A_C \cos(2\pi f_c t + \phi(t)) \qquad (2.8)$$

Where

$$\phi(t) = \begin{cases} k_p m(t) & \leftarrow \text{PM} \\ 2\pi k_f \int_0^t m(c)dc & \leftarrow \text{FM} \end{cases} \qquad (2.9)$$

$\downarrow$

$$\frac{d\phi(t)}{dt} = \begin{cases} k_P \frac{d}{dt} m(t) & \leftarrow \text{PM} \\ 2\pi k_f m(t) & \leftarrow \text{FM} \end{cases} \qquad (2.10)$$

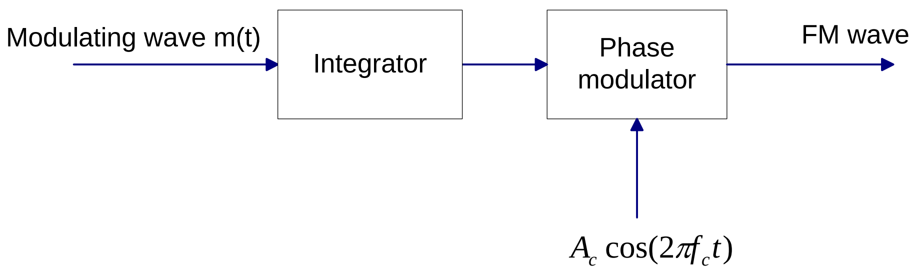
From equation 2.8 it is clear that the envelope of a PM or FM signal is constant (equal to the carrier amplitude).

Where as the envelope of an AM signal is dependent on the message signal.
Comparing equation 2.4 (page118) with 2.7(page119) reveals that an FM signal may be

regarded as a PM signal in which the modulating wave is :

$\int_0^t m(\tau)d\tau$ in place of $m(t)$

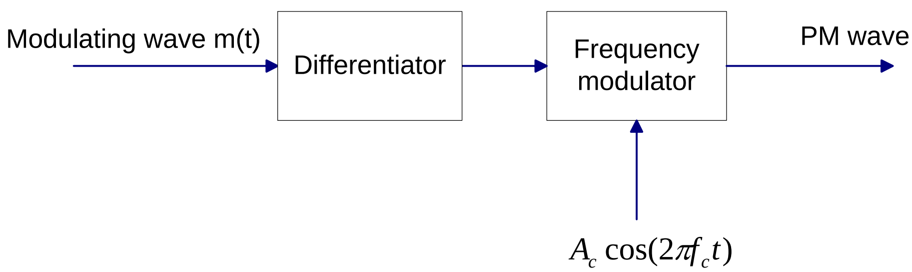
This means that an FM signal can be generated by first integrating $m(t)$ and the using the result as the input to a phase modulator (see Fig 2.1).



(Scheme for generating an FM wave)

Fig 2.1

Conversely, a PM signal can be generated by first differentiations m (t) and then using the result as the input to a frequency modulator as shown Fig 2.2.



(Scheme for generating a PM wave)

Fig 2.2.

Scheme for generating PM wave (Fig2.2)

We may thus deduce all the properties of PM signal from those of FM signal or vice versa.

$\therefore$ We concentrate our attention on FM signals.

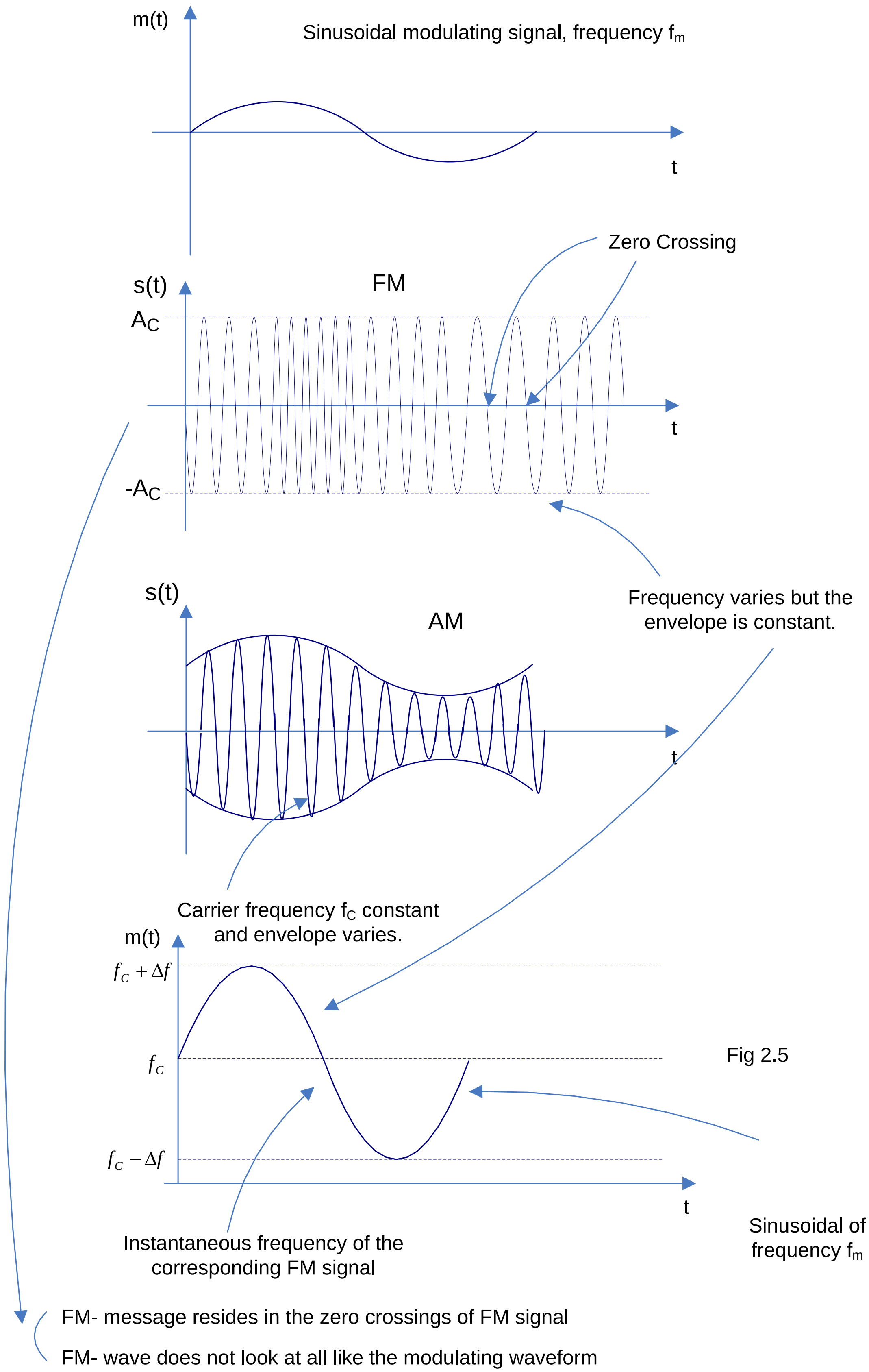

m(t)
Sinusoidal modulating signal, frequency fm
t
Zero Crossing
s(t)
FM
AC
-AC
t
Frequency varies but the envelope is constant.
s(t)
AM
t
Carrier frequency fC constant and envelope varies.
m(t)
fC + Δf
fC
fC − Δf
t
Instantaneous frequency of the corresponding FM signal
Sinusoidal of frequency fm
FM- message resides in the zero crossings of FM signal
FM- wave does not look at all like the modulating waveform

Fig 2.5

Frequency modulation

The FM signal s(t) defined by equation 2.7(page 119) $s(t) = A\cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$

Is a nonlinear function of the modulating signal m(t), which makes frequency modulation a nonlinear modulation process.

Consequently, unlike amplitude modulation, the **spectrum** of an FM signal is not related in a simple manner to that of a modulating signal –its analysing is much more different than that of an AM signal.

We propose two simple cases for the spectral analysis of an FM signal:

(1) A single tone modulation that produces a narrow FM signal.
(2) A single tone modulation that produces wideband FM signal.

Consider a sinusoidal modulating signal, $m(t) = A_m \cos(2\pi f_m)$ (2.11).
The instantaneous frequency of the resulting FM signal is given by:

$f_i(t) = f_c + k_f m(t)$ ← equation (2.5)

$f_i(t) = f_c + k_f A_m \cos 2\pi f_m t$ (2.12).

$$f_i(t) = f_c + \Delta f \cos 2\pi f_m t \qquad (2.13).$$

The quantity $\Delta f = k_f A_m$ is called the **'frequency deviation'**, representing the **maximum departure** of the instantaneous frequency of the FM signal from the carrier frequency f_c.

$$\Delta f = k_f A_m \qquad (2.14).$$

A fundamental characteristic of an FM signal is that the frequency deviation Δf is proportional to the amplitude of the modulating signal and is independent of the modulation frequency.

The angle $\theta_i(t)$ of the FM signal,

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau \qquad (\text{See } 2.6)$$

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f A_m \frac{\sin \omega_m t}{\omega_m}$$

$$\theta_i(t) = 2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t)$$

$$\boxed{\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)} \qquad (2.15).$$

The ratio of the frequency deviation Δf to the modulation frequency f_m is commonly called the **modulation index** of the FM signal.

$$\beta = \frac{\Delta f}{f_m} \qquad (2.16).$$

$$\Rightarrow \theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$$

β is measured in radians.

The FM signal is given by: $s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)$ (2.17).

Depending on the value of the modulation index β, we may distinguish two cases of frequency modulation:

- **Narrowband FM,** for which β is small compared to one radian.
- **Wideband FM,** for which β is large compared to one radian.

Narrowband Frequency Modulation:

Consider equation 2.17 above,

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) =$$
$$A_c \cos 2\pi f_c t\,.\cos(\beta \sin(2\pi f_m t)) - A_c \sin 2\pi f_c t\,.\ \sin(\beta \sin 2\pi f_m t) \qquad (2.18)$$

β Is small compared to one radian, we may approximate

$$\cos(\beta \sin 2\pi f_m t)) \approx 1$$

and

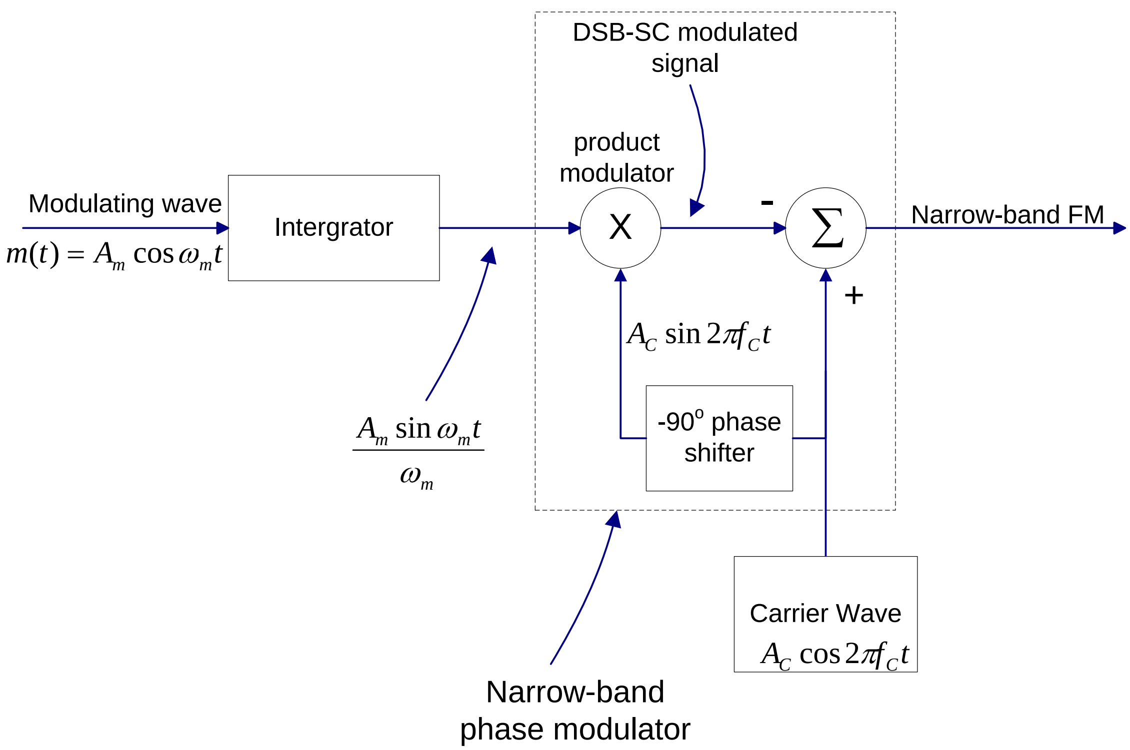
$$\sin(\beta \sin(2\pi f_m t) \approx \beta \sin(2\pi f_m t)$$

Hence equation2.18 becomes

$$s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t).\sin(2\pi f_m t) \qquad (2.19)$$

Narrowband FM signal

Equation 2.19 can be implemented as follows (Fig2.4).



Method for generating narrowband FM signal.

Fig 2.4.

Ideally, an FM signal has a **constant envelope** and for the case of a sinusoidal modulating frequency f f_m, the angle $\theta_i(t)$ is also sinusoidal with the same frequency. (See page122 Fig2.3).

However the modulated signal produced by the narrowband modulator of Fig 2.4 (page 127) differs from this ideal condition in two fundamental respects:

- The envelope contains residue amplitude modulation and therefore varies with time.
- For a sinusoidal modulating the angle $\theta_i(t)$ contains harmonic distortion in the form of third and higher-order harmonic of the modulation frequency f_m.

However, if β(modulation index) is restricted to $\beta \leq 0.3$ radians, the effect of residual AM and harmonic are limited to negligible levels.

Returning to equation 2.19(pp.127),

$$s(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t).\sin(2\pi f_m)t$$

Narrowband FM signal

$$s(t) = A_c \cos 2\pi f_c t + \frac{1}{2}\beta A_c \left[\cos 2\pi(f_c + f_m)t - \cos 2\pi(f_{c-} f_m)t\right] \qquad (2.10)$$

Narrowband FM signal

Consider an AM signal equation (page26, equation12)

$$s(t) = A\cos(2\pi f_c t) + \frac{MA_c}{2}\left[\cos 2\pi(f_c + f_m)t + \cos 2\pi(f_c - f_m)t\right] \qquad (2.11)$$

$\mu -$ modulation factor of AM

AM signal

Comparing equation 2.10$ 2.11, we see that in the case of sinusoidal modulation, the Basic difference between an AM signal $ a narrowband FM signal is that the algebraic sign of the lower side frequency in the narrowband FM is reversed.
Thus, a narrowband FM signal requires essentially the same transmission band width (i.e.2 f_m) as the AM signal.

Wideband Frequency Modulation:

The FM signal itself is given by

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

We wish to determine the spectrum of the single tone FM signal above, for an arbitrary value of the modulation index β.

Assuming $\beta > 1$ (wideband frequency modulation we may write the above FM signal as

$$\mathrm{s(t) = A_c}\left[\cos \mathrm{w_c} t . \cos \beta \sin \mathrm{w_m} t) - \sin \mathrm{w_c} t . \sin(\beta \sin \mathrm{w_m} t)\right]$$

- S(t) is no periodic unless f_c is an integral multiple of $f_{m.}$
- We assume that f_c is large enough compared to the bandwidth of the FM signal.

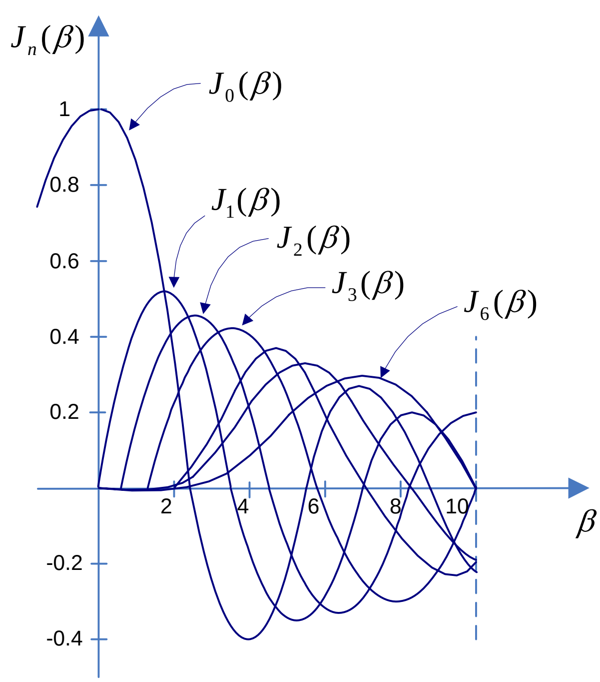
We know that:

$$\cos(\beta \sin \omega_m t) = \mathrm{J}_0(\beta) + \sum_{n \text{ even}}^{\infty} 2 J_n(\beta) \cos n\omega_m t \qquad (2.13)$$

$$\text{Sin } (\beta \sin \omega_m t) = \sum_{n \text{ odd}}^{\infty} 2 J_n(\beta) \sin n\omega_m t \qquad (2.14)$$

Where n is positive and $J_n(\beta)$ are coefficient of Bessel functions of the first kind, of order n argument β.

Figure below shows the Bessel function $J_n(\beta)$ versus modulation index β for different positive integer value of n.



Bessel function for n=0 to n=6

We can develop further insight into the behaviour of the Bessel function $J_n(\beta)$,

(1) $J_n(\beta) = (-1)^n J_{-n}(\beta)$ For all n both positive & negative.

(2) For small values of the modulation index β, we have

$$J_0(\beta) = 1$$

$$J_1(\beta) = \frac{\beta}{2}$$

$$J_n(\beta) = 0 \quad n > 2$$

(3) $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

Substituting equations 2.13 & 2.14 into equation2.12 and expanding products of sines and cosines finally yields:

$$s(t) = A_c J_0 \beta \cos \omega_c t$$
$$+ \sum_{nodd}^{\infty} A_c J_n(\beta)[\cos(\omega_c + n\omega_m)t - \cos(\omega_c - n\omega_m)t]$$
$$+ \sum_{neven}^{\infty} A_c J_n(\beta)[\cos(\omega_c + n\omega_m)t + \cos(\omega_c - n\omega_m)t]$$

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t \qquad (2.15)$$

↑

Wide-band FM

s(t) is the desired form for the Fourier series representation of the single tone FM signal s(t) for an arbitrary value of β.

The discrete spectrum of s(t) is obtained by taking the Fourier transform of both sides of equation 2.15 & we have:

$$s(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - (f_c + nf_m)) + 5(f + (f_c + nf_m))] \qquad (2.16)$$

↑

FM signal

From 2.15& 2.16 we may make the following observation:

(1) The spectrum of an FM signal contain a carrier component and an infinite set of side frequencies located symmetrically on either side of the carrier at frequency separations f $f_m, 2f_m, 3f_m$, ……………………..(note in AM system a sinusoidal modulating signal given rise to only one pair of side frequencies.

(2) For the special case of β small compared to unity ,only the Bessel coefficients $J_0(\beta)$ and $J_1(B)$ have significant values , so that the FM signal is effectively composed of a carrier and a single pair of side frequency at $f_c \pm f_m$.

This situation corresponds to the special case of narrowband FM that was considered earlier.

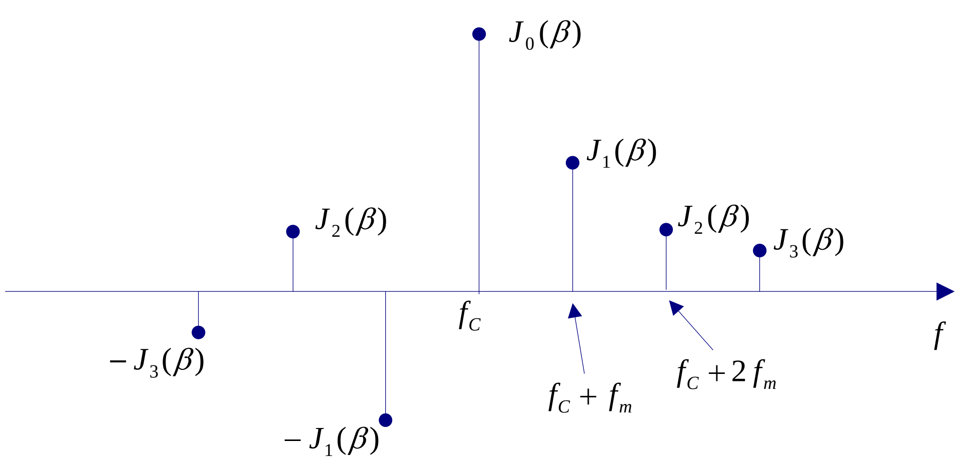
$$s(t) = \frac{AC}{2} J_0(\beta)[\delta(f - f_c) + \delta(f + f_c)] + \frac{AC}{2} J_1(\beta)[\delta(f - (f_c + f_m)) + \delta(f + f_c + f_m))]$$

n=0 n=1

[Note: $J_0(\beta) = 1$, $J_1(\beta) = \frac{\beta}{2}$]

Note: Line spectrum of FM with tone modulation

$$J_{-n}(\beta) = (-1)^n J_n(\beta)$$



Note : $J_0(\beta) = 0$ when $\beta = 2.4$ & 5.5 (Bessel function) i.e. carrier line has zero amplitude.

(3) The amplitude of the carrier component varies with β according to $J_0(\beta)$. Unlike AM signal, the amplitude of FM carrier component of an FM signal is dependent the modulation an index β.

The envelope of an FM signal is constant, so that the average power of such a signal developed across 1 ohm resistor is also constant.

$$\therefore \quad P = \left(\frac{AC}{\sqrt{2}}\right)^2 = \frac{1}{2}AC^2$$

or

$$P = \sum_{n=-\infty}^{\infty}\left(\frac{AC}{\sqrt{2}}\right)^2 . J_n^2(\beta) = \frac{AC^2}{2} \underbrace{\sum_{n=-\infty}^{\infty} J_n^2(\beta)}_{=1}$$

$$\boxed{\therefore P = \frac{AC^2}{2}} \qquad (2.17)$$

We wish to investigate the ways in which variation in the **amplitude** and **frequency** of a **sinusoidal** affect the spectrum of the FM signal.

Consider the case when the frequency of the modulating signal is fixed, but amplitude is varied.

$$m(t) = A_m \cos(\omega_m t)$$

A_m: varied; ω_m: fixed

$$\Delta f = k_f A_m \qquad \beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

Δf: frequency deviation

Figure below shows amplitude spectrum of FM signal for $\beta = 1, 2$ and 5

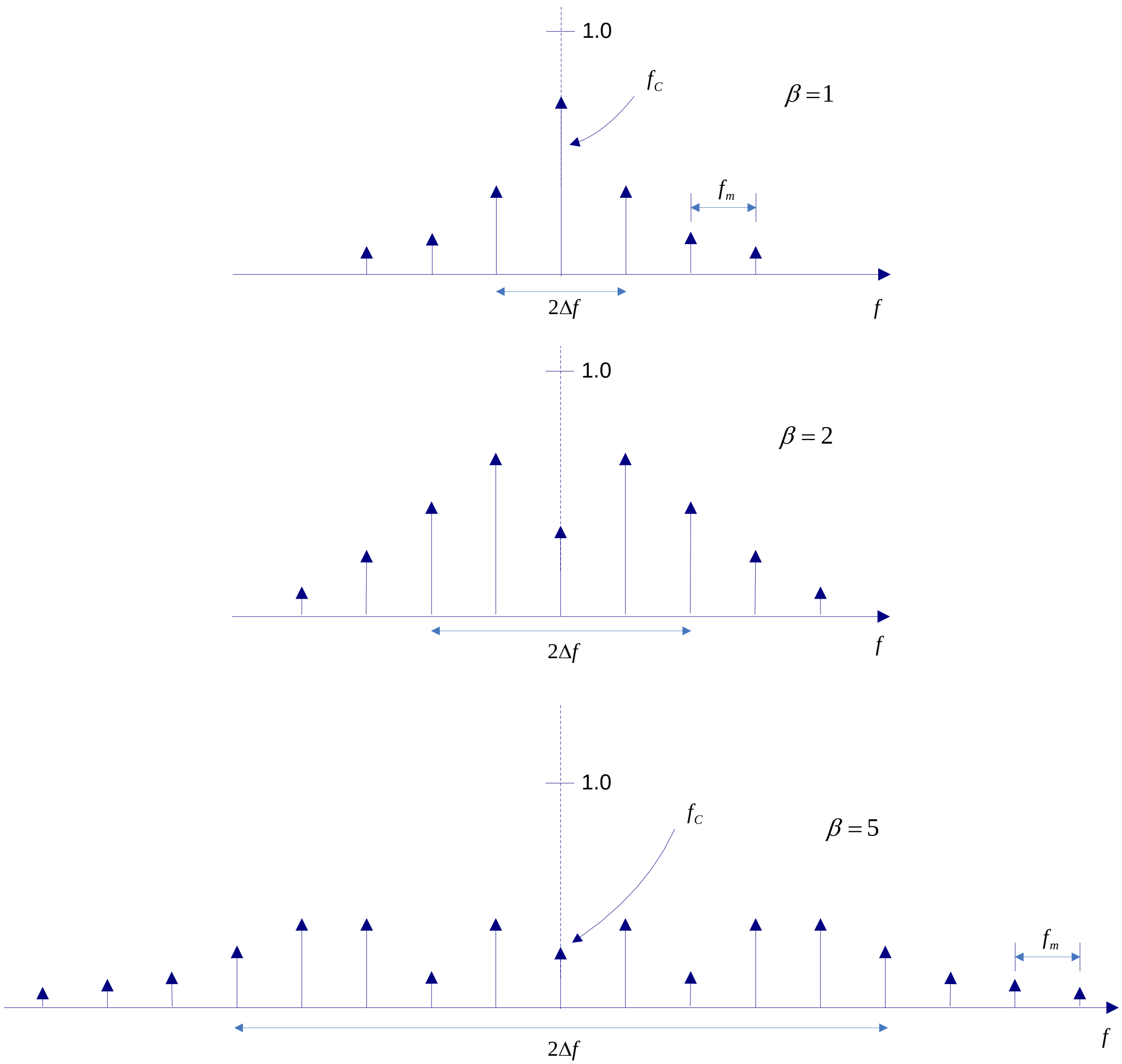


Fig 2.6

Consider next the case when the amplitude of the modulating signal is tuned : That is the frequency deviation Δf is maintained constant.

$$m(t) = A_m \cos(\omega_m t)$$

fixed (A_m), varied (ω_m)

$$\Delta f = K_f A_m$$

fixed

$$\beta = \frac{\Delta f}{f_m}$$

The amplitude spectrum of the resulting FM signal is shown is Fig2.7.

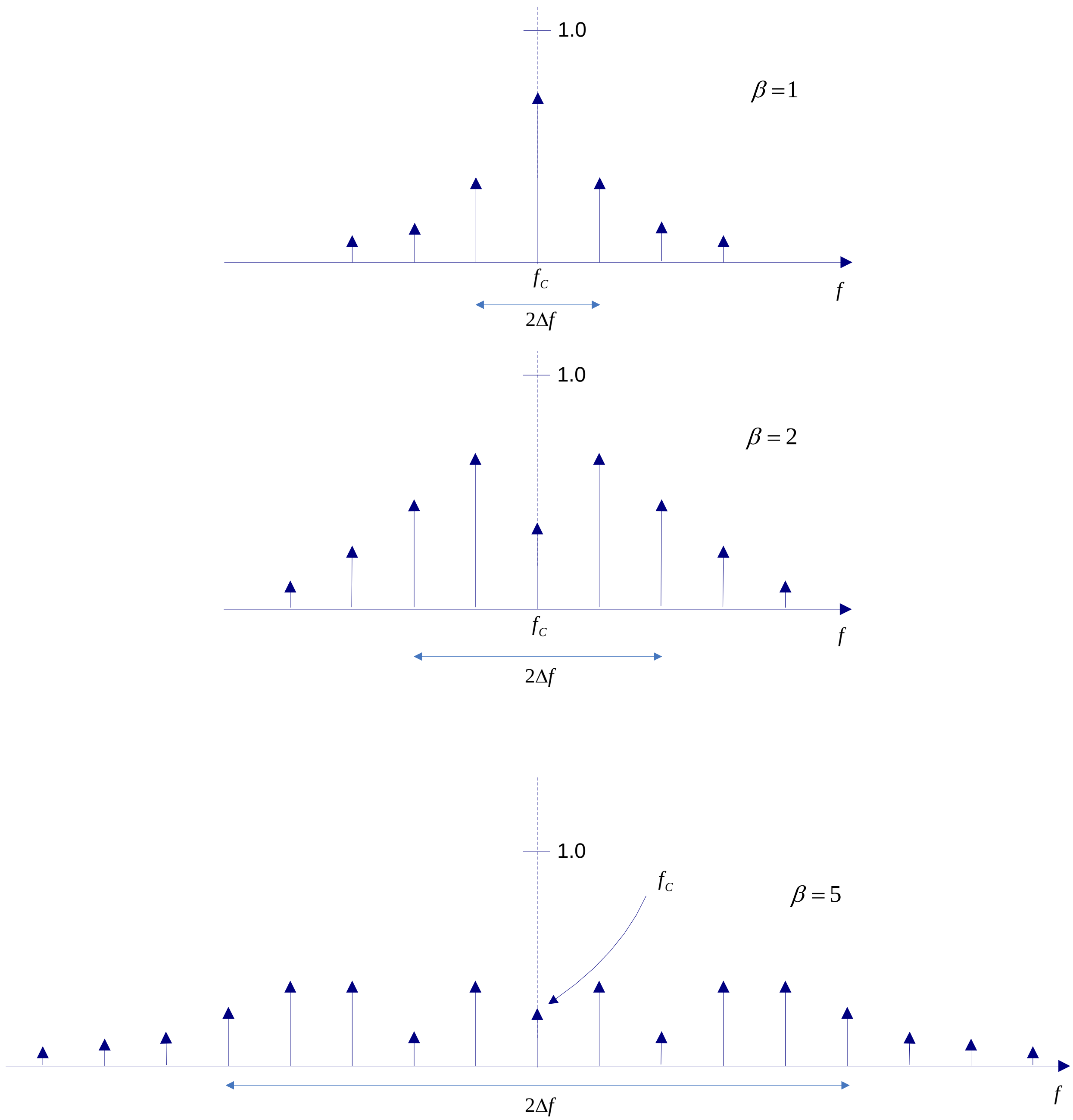


Fig 2.7

We see that when Δf is fixed and β is increased we have an increasing number of spectral lines crowding into the fixed frequency interval.

$f_c - \Delta f < |f| < f + \Delta f$

That is, when β approaches infinity, the bandwidth of the FM wave approaches the limiting value of $2\Delta f$.

Transmission Bandwidth of FM signals:

In theory an FM signal contains an infinite number of side frequencies so that the band width required to transmit such a signal is infinite in extent.

In practice, we find that FM signal is effectively limited to a finite number of significant side frequencies.

In the case of an FM signal generated by a single tone modulating wave of frequency f_m , The side frequencies that are separated from the carrier frequency f_C by an amount greater than the frequency deviation Δf decrease rapidly toward zero.

Specifically for **large values of β ,** the bandwidth approaches, and is only slightly greater than the total frequency **deviation** $2\Delta f$ (See Fig 2.6).

On the other hand, for small values of β , the spectrum of FM signal is effectively limited to the carrier frequency $f_C \pm f_m$ so that the brand width **approaches** $2f_m$.

We may thus define an approximate rule for the transmission bandwidth for an FM signal generated by a single tone modulating signal f_m as follows:

$$\Delta f = \beta f_m$$

$$\beta_T \approx 2\Delta f + 2f_m = 2\Delta f + \frac{2\Delta f}{\beta}$$

Transmission Bandwidth

$$\boxed{\beta_T = 2\Delta f\left(1 + \frac{1}{\beta}\right)}$$

Carson's rule

Note: for large β , $\beta_T = 2\Delta f(1+0) = 2\Delta f$

Consider next the move general case of an arbitrary modulating signal m (t) with its **highest frequency component** donated by W.

The bandwidth required to transmit an FM signal generated by this modulating signal is estimated as follows:

First we determine the **deviation ratio D** defined as the ratio of the frequency deviation Δf which corresponds to the maximum possible amplitude of the modulating signal m(t), to the highest modulation frequency W.

$$D = \frac{\Delta f}{W}$$

Deviation ratio

The deviation ratio D plays the same role for non sinusoidal modulation that the modulation index β plays for the case of sinusoidal modulation.

Example:

The maximum value of frequency deviation Δf is normally fixed at 75KHz for commercial FM broadcasting by radio.

If we take the modulation frequency W=15KHzwhich is typically the 'maximum' audio frequency of interest in FM transmission, we find that:

$$D = \frac{\Delta f}{W} = \frac{75}{15} = 5$$

Using the transmission bandwidth equation $\beta_T = 2\Delta f + 2 f_m$

$$\beta_T = 2\Delta f\,(1 + \frac{1}{\beta})$$ (page…)

and replacing β by D and replacing f_m by W the approximate value of the transmission bandwidth of the FM signal is obtained as

$$\beta_T = 2(75+15) = 180 KHz$$

In practice, a bandwidth of 200 KHz is allocated to each transmitter.

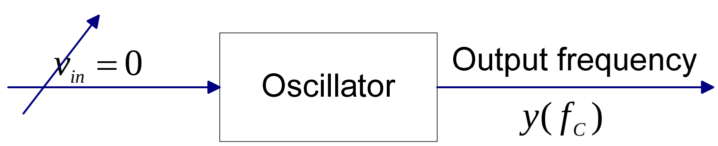
Generation of FM signals

There are essentially two basic methods of generating frequency –modulated signals namely,

- **Direct FM**
  (Carrier frequency is directly varied in accordance with the input base band signal, which is readily, accomplished using a voltage controlled oscillator (VCO).

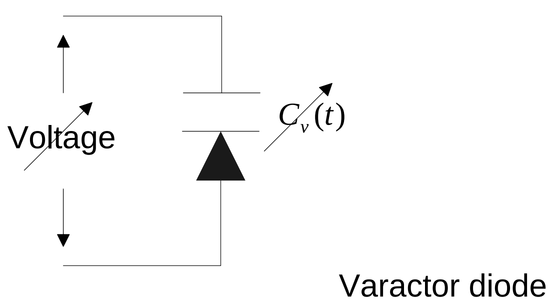
- **Indirect FM**
  (The modulating signal is first used to produce a narrowband FM signal and frequency multiplication is next used to increase the frequency deviation to the desired level).

One method for generating an FM signal directly is to design an oscillator whose Frequency changes with the input voltage.
When the input voltage is zero, the oscillator generating a sinusoid with frequency f_C and when the input voltage changes, this frequency changes accordingly.

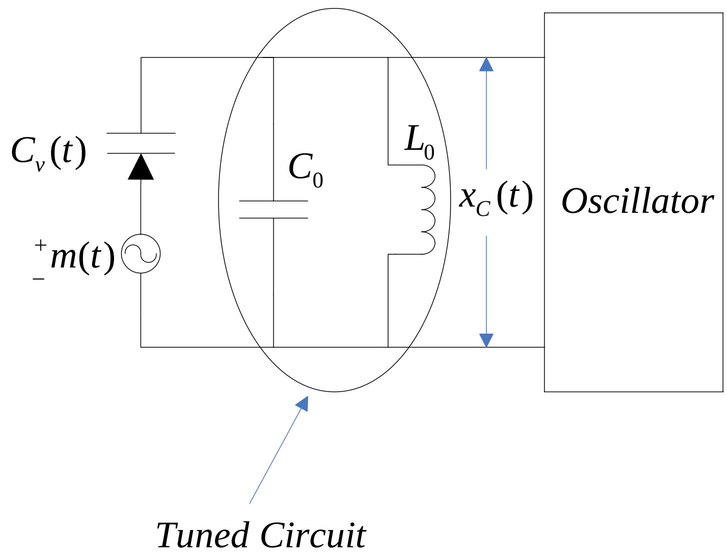


Voltage Controlled Oscillator (VCO)

In this approach a "Varactor diode" is used. A Varactor diode is a capacitor whose capacitance changes with the applied voltage.



Therefore , if this capacitor is used in a tuned circuit of the oscillator and the message signal is applied to it, the frequency of the tuned circuit ,and the oscillator will change in accordance with the message signal(see diagram below).



Let the indicator in the tuned circuit be L_0 and the capacitance of the varactor diode is given by

$$c(t) = c_0 + k_0 m(t) \qquad (2.18)$$

When m (t) =0, the frequency of the tuned circuit is given by:

$$f_c = \frac{1}{2\pi\sqrt{L_0 C_0}}$$

In general if m (t) $\neq$ 0, we have

$$f_i(t) = \frac{1}{2\pi\sqrt{L_0(c_0 + k_0 m(t))}}$$

c(t) (2.8)

$$\therefore\ f_i(t) = \frac{1}{2\pi\sqrt{L_0 C_0}} \cdot \frac{1}{\sqrt{1 + \frac{h_0}{C_0} m(t)}} = f_c \cdot \frac{1}{\sqrt{1 + \frac{k_0}{C_0} m(t)}}$$

Assuming that $\in = \frac{h_0}{c_0} m(t) << 1$

and using the approximations

$$\sqrt{1+\varepsilon} = (1+\varepsilon)^{\frac{1}{2}} \approx 1 + \frac{\varepsilon}{2}$$

$$\frac{1}{1+\varepsilon} \approx 1 - \varepsilon$$

$$\varepsilon < 1$$

We obtain

$$f_i(t) = f_c \cdot \left[1 - \frac{k_0}{2c_0} m(t) \right] \qquad (2.19)$$

This is the relation for a frequency modulated signal.

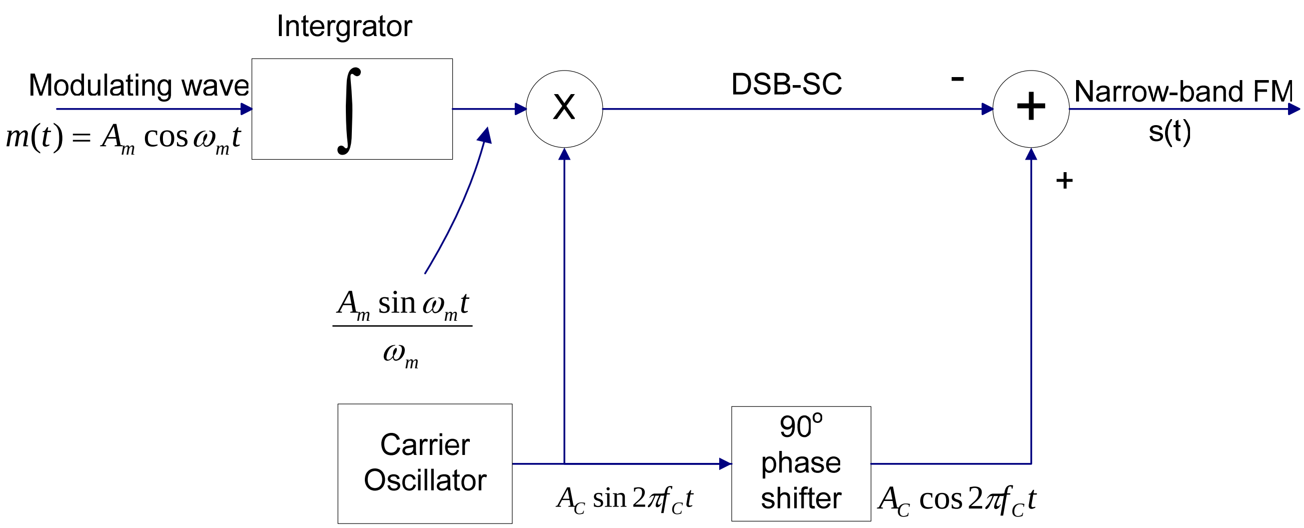
$$x_c(t) = A_c \cos\theta_i(t) = A_c \cos(2\pi f_i(t))$$

where $f_i(t) = f_c \left[1 - \frac{k_0}{2c_0} m(t) \right]$

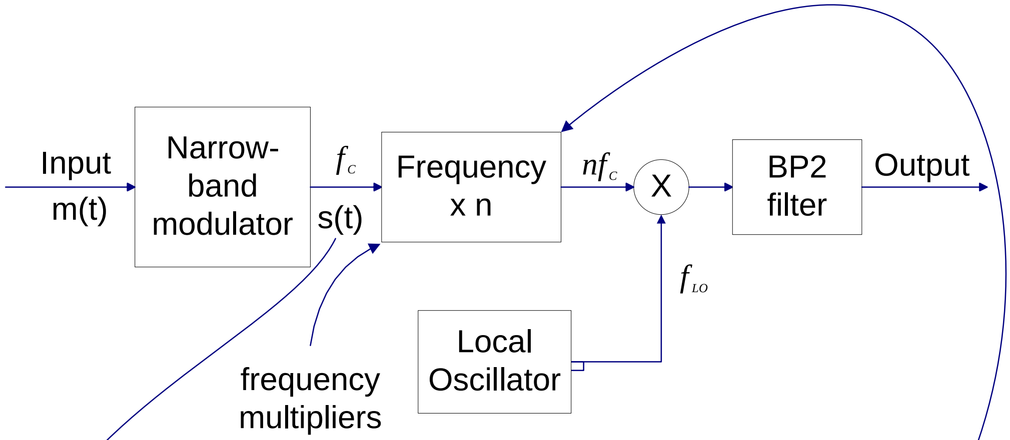
<u>Indirect method for generating of FM</u>

Another approach for generating an angle modulation signal is to first generate a **narrowband** angle –modulated signal, and then change it to a wideband signal. Due to the similarity of conventional AM signals, generation of narrowband angle modulated signals is straightforward.

Generation of narrow-band angle modulated signal (see page … equation 2.19)

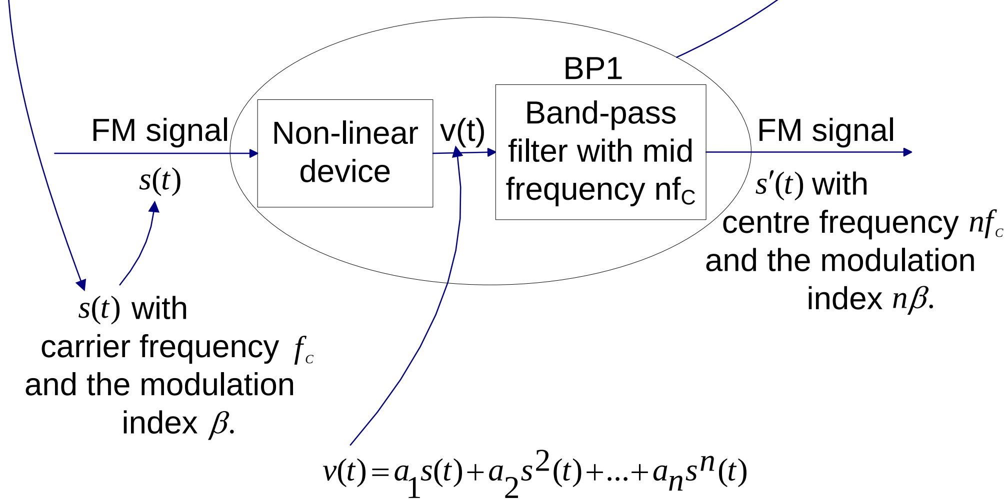


The next step is to use the narrowband angle modulated signals to generate a wideband modulated signals (see diagram below).



The narrow-band angle modulated signal enters a frequency multiplier that multiplies the instantaneous frequency of the input by some constant n.

The frequency multiplier consists of a nonlinear device followed by a band pass filter (see below).



Where $a_1, a_2, a_{3,} ......, a_n$ coefficient and n are is the highest order of non linearity.

The input signal (FM) is defined by

$$s(t) = A\cos(2\pi f_C t + 2\pi k_f \int_0^t m(\tau)d\tau)$$

Whose instantaneous frequency is:

$$f_i(t) = f_C + k_f m(t)$$

The mid-band frequency of the band-pass filter is set to nf_C where f_C is carrier frequency of the incoming FM single s(t).

The band-pass filter is designed to have a bandwidth equal to n times the transmission bandwidth of s(t).
After band-pass filtering the nonlinear devices output v(t), we have a new FM signal defined by:

$$s'(t) = A'_C \cos\left[2\pi n f_C t + 2\pi n k_f \int_0^t m(\tau)d\tau\right]$$

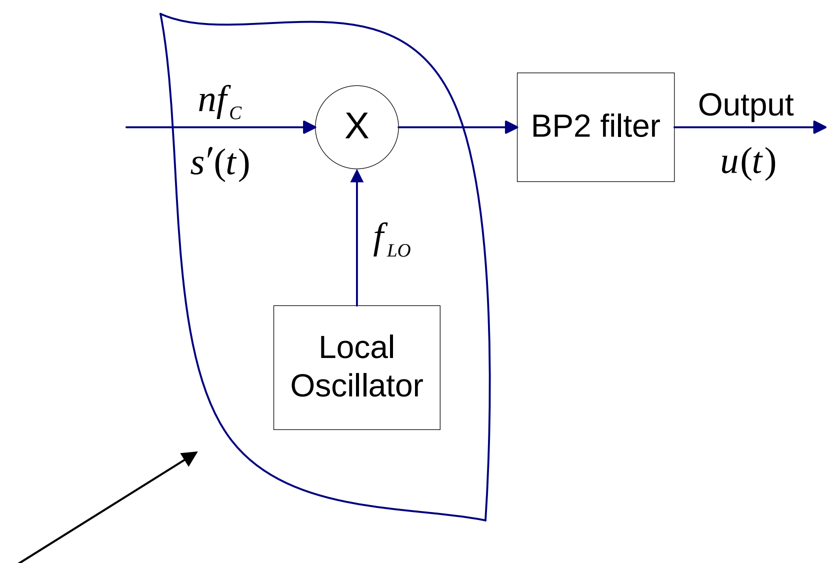
Whose instantaneous frequency is:

$$f_i'(t) = nf_C + nk_f m(t) \qquad (2.20)$$

Comparing $f_i(t) = f_c + k_f m(t)$ and $f_i'(t) = nf_C + k_f nm(t)$,
we see that nonlinear processing circuit in page … acts as a frequency multiplier.
The frequency multiplication ratio is determined by the highest power n in the equation

$$v(t) = a_1 s(t) + a_2 s^2(t) + ... + a_n s^n(t)$$

Note: see the top diagram in page … after the above described process , there is no guarantee that the carrier frequency of this signal n f_C will be the desired carrier frequency, we may perform an up or down conversion to shift the modulated signal to the desired center frequency, (see page…). This stage consists



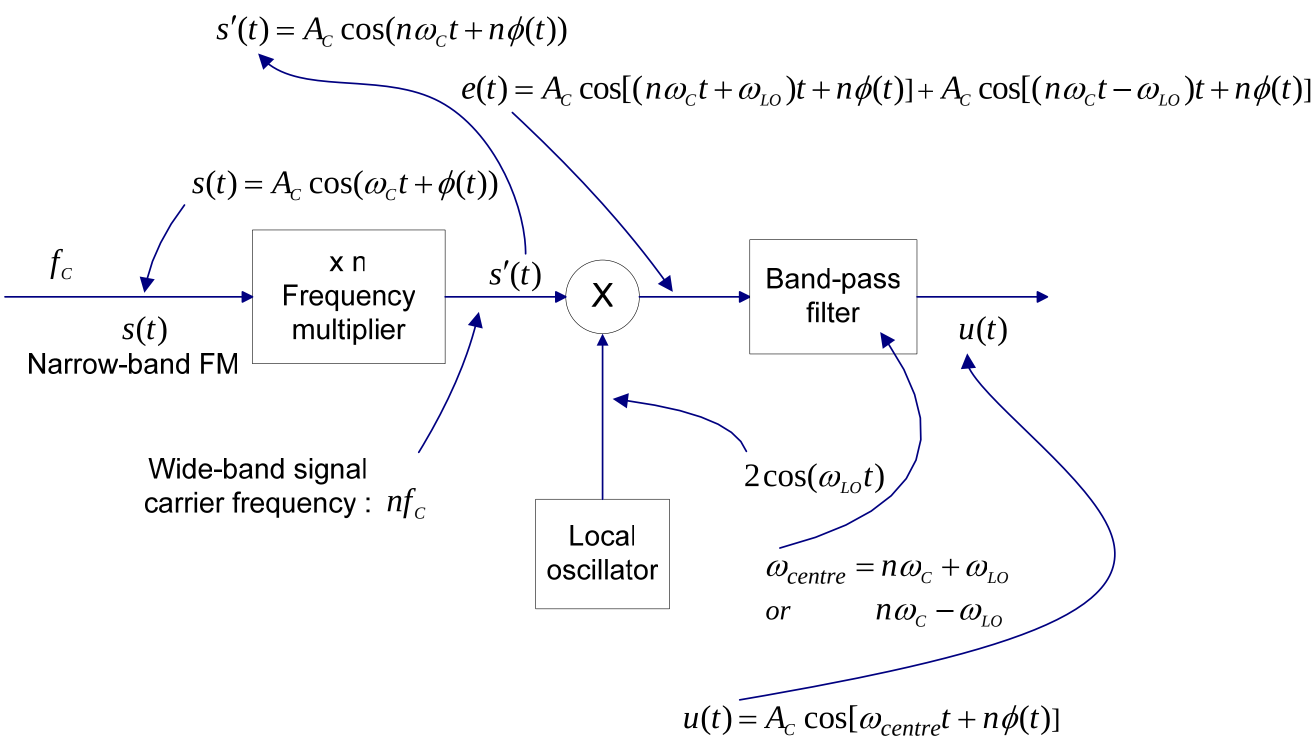
of a mixer and a band-pass filter (BP2). If the frequency of the local oscillator of the mixer is f_{LO} and we are using a down converter, the final wideband FM signal is given by

$$u(t) = A'_C \cos\left[(2\pi(nf_C - f_{L0})t + 2\pi n k_f \int_0^t m(\tau)d\tau\right] =$$

$$A'_C \cos(2\pi(nf_C - f_{L0})t + n\phi(t))$$

Since we can freely chose n and f_{L0}, we can generate any modulation index at any desired carrier frequency by this method.

Example: a narrowband to wideband converter is implemented as follows.



The output of the narrowband frequency modulator is given by:

$$s(t) = A_c \cos(w_c t + \phi(t))$$

With $w_c = 2\pi \times 10^5 Hz$, the peak frequency deviation of $\phi(t)$ is 50Hz and the bandwidth of $\phi(t)$ is 500Hz. The wideband output $u(t)$ is to have a carrier frequency of 85MHz and a deviation ratio of 5. Determine the frequency multiplier factor n. Also determine two possible local oscillator frequencies. Determine the centre frequency and the bandwidth of the band-pass filter.

Deviation ratio at the output of the narrowband FM (i.e. s (t)):

$$D = \frac{\Delta f}{W} = \frac{50Hz}{500Hz} = 0.1 \qquad \left.\begin{matrix} \Delta f = 50Hz \\ W = 500Hz \end{matrix}\right\} \text{given}$$

The frequency multiplier n is:

$$\text{n} = \frac{D \text{at the output}}{\text{D}} = \frac{5}{0.1} = 50$$

Wideband carrier frequency =n. w_c =$50 \times 10^5 = 5MHz$

We need a carrier of 85MHz

$\therefore w_{LO} = 85 + 5 = 90MHz$

or

$w_{\text{LO}} = 85 - 5 = 80MHz$

Centre frequency of the BP filter must be equal to the desired carrier frequency of the wideband output .i.e85MHz.

The BW of the band pass filter is calculated using Carson's rule.

$$B = 2\Delta f + 2w = 2w(\frac{\Delta f}{w} + 1) = 2 \times 500(5 + 1) = 6KHz$$

Demodulation of FM:

The demodulation of A FM signal requires a circuit that yields an output voltage that varies linearly proportional to the frequency deviation of the input.
Such circuits are known as **discriminators**.

There are many different circuit designed for frequency detection (by a frequency detector –known as a discriminator).

There are four operational categories:

- ✓ FM to AM conversion
- Phase shift discrimination
- Zero crossing detection
- ✓ Frequency feedback(PLL)

FM to AM conversion

Any device or circuit whose output equals the time derivation of the input produces FM to AM conversion

Let $s(t) = A_C \cos\theta_i(t)$

With $\theta_i(t) = 2\pi f_C t + 2\pi k_f \int_0^t m(\tau)\tau$

$\therefore \dot{\theta}_i(t) = 2\pi f_C + 2\pi k_f m(t)$

$$\dot{\theta}_i(t) = 2\pi[f_C + k_f.A_m \cos 2\pi f_m t]$$

$$= 2\pi[f_C + \Delta f \cos 2\pi f_m t]$$

$$\dot{s}(t) = -A_C \sin\theta_i(t).\dot{\theta}_i(t) \qquad [s(t) = A_C \cos\theta_i(t)]$$

$$\dot{s}(t) = -A_C \sin\theta_i(t).2\pi[f_C + \Delta f \cos 2\pi f_m t]$$

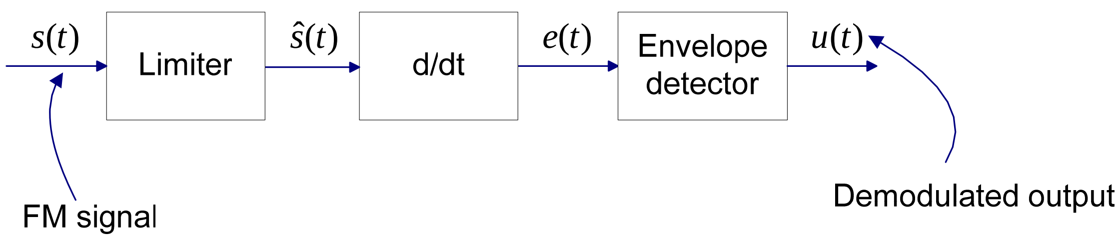
$$\dot{s}(t) = 2\pi A_C[f_C + \Delta f \cos\omega_m t]\ .\sin(\theta_i(t) \pm 180^\circ) \qquad (2.21)$$

AM envelope

Hence an envelope detector with input $\dot{s}(t)$ yields an output

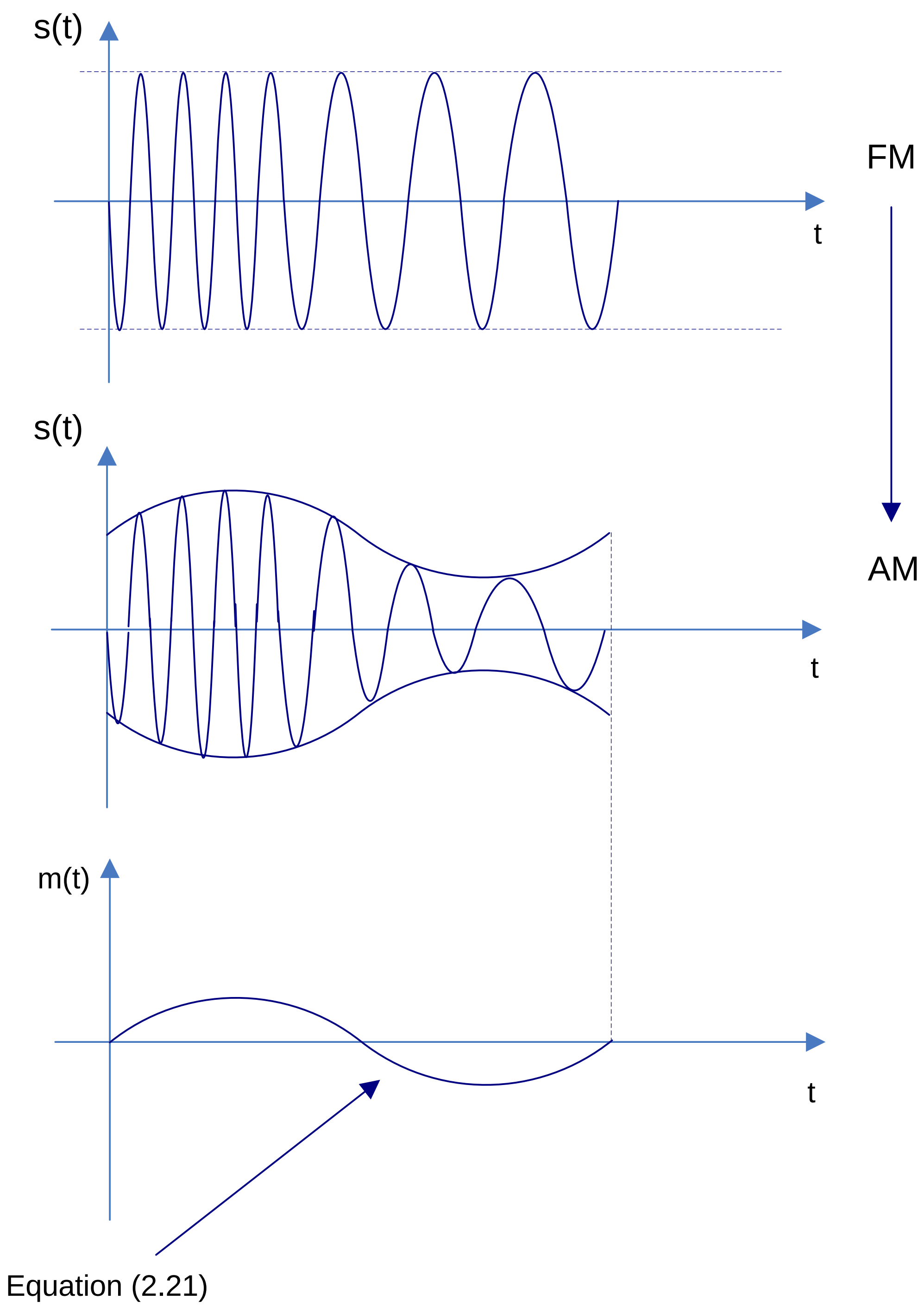
proportional to $f_i(t) = f_C + \Delta f \cos\omega_m t.$

Figure below shows a frequency detector based on equation(2.21).



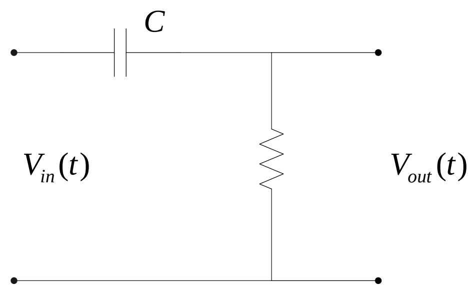
The limiter at the input removes any spurious amplitude variations (due to noise) from s(t) before reaching the envelope detector.

Typical waveforms are shown below:



FM to AM conversion & demodulated output

A differentiator can be implemented using an RC network.

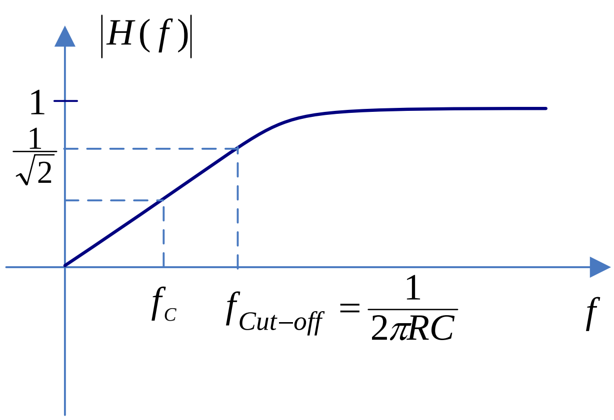


$$\frac{V_o(s)}{V_i(s)} = H(s)$$

Transfer function

$$H(f) = \frac{R}{R + \frac{1}{j2\pi f C}} = \frac{j2\pi f RC}{1 + j2\pi f RC}$$

The amplitude response of $H(f)$ is shown below:



High-pass filter (Differentiator)

If all frequencies present in the input are low so that

$$f << \frac{1}{2\pi RC} \Rightarrow 2 f \pi RC << 1$$

then the transfer function can be approximated by:

differentiator

$$H(f) = j2\pi fRC = j\omega RC$$

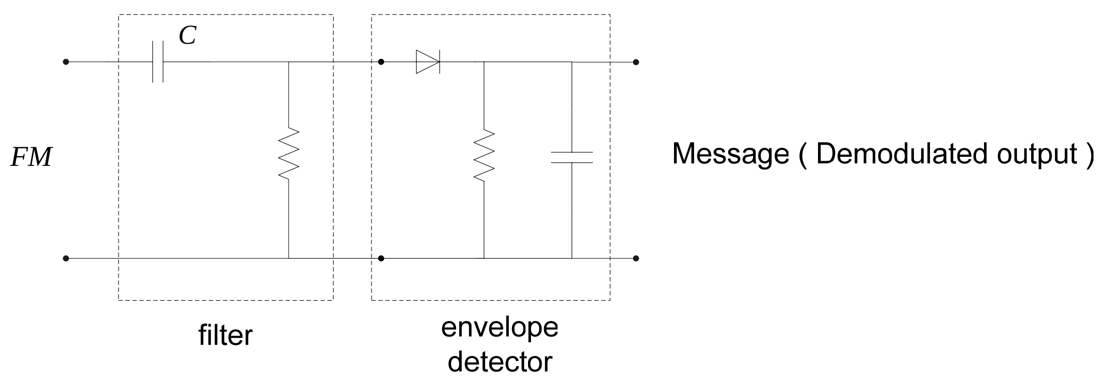
$$|H(f)| = 2\pi RCf \quad \text{i.e. } |H(f)\propto f|$$

For small f , the RC network has linear amplitude frequency characteristic required of an ideal discriminator.

$$H(f) = j\omega RC$$

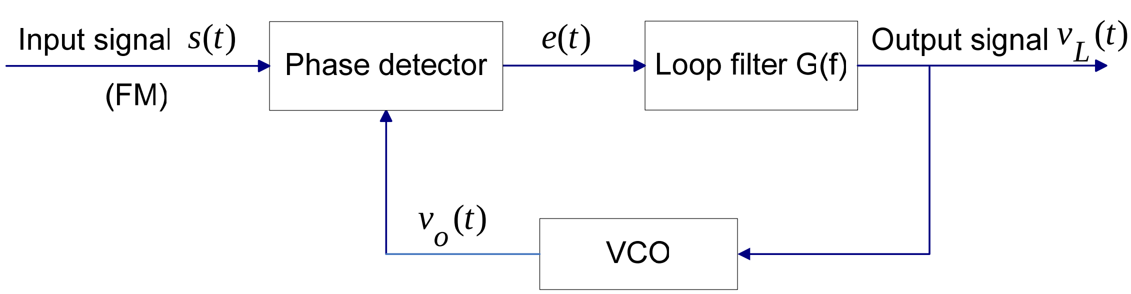
If small f , the RC filter acts as a differentiator with gain RC.
Thus the RC network can be used in place of differentiator in page ….

A simple discriminator circuit is shown below:



FM demodulator using a phase locked loop (PLL):

For PLL see page…



The input to the PLL is the angle modulated signal $s(t) = A_C \cos[2\pi f_C t + \phi(t)]$, where for FM $\phi(t) = 2\pi k_f \int_0^t m(\tau)d\tau$

The VCO generates a sinusoidal of a fixed frequency $f_{C,}$ in the absence of an input control voltage. $[i.e. V_L(t) = 0]$

The instantaneous frequency of the VCO is

$$f_i(t) = f_C + k_v v_L(t)$$

Where k_v is a deviation constant with units Hz/Volt, consequently the VCO output may be expressed as

$$v_o(t) = A_o \sin(2\pi f_C t + \phi_o(t))$$

Where $\phi_o(t) = 2\pi k_v \int_0^t v_L(\tau)d\tau$

The phase detector is basically a multiplier and the filter that rejects the signal component centered at 2 f_C . Hence its input may be expressed as

$$e(t) = \frac{1}{2} A_C A_0 \sin[\underbrace{\phi(t) - \phi_0(t)}_{=\phi_e(t)}]$$

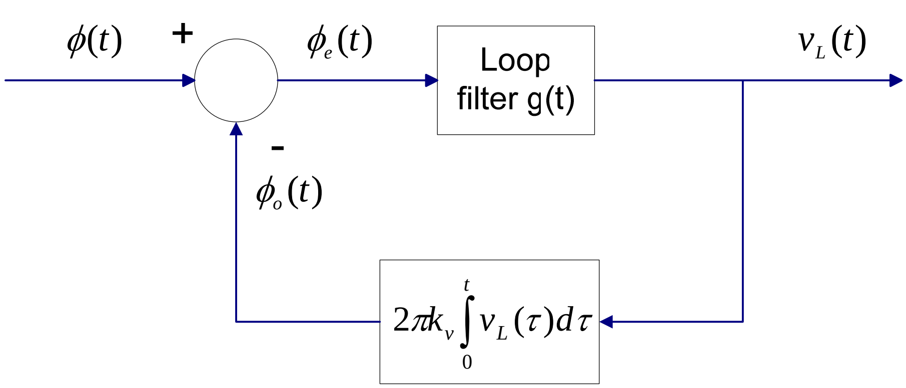
Let us assume that the PLL is in lock, so that the phase error is small, then

$$\sin \phi(t) - \phi_o(t) \approx \phi(t) - \phi_o(t) = \phi_e(t)$$

We may express the phase error as:

$$\phi_e(t) = \theta(t) - 2\pi k_v \int_0^t V_L(\tau)d\tau$$

Using this equation we obtain a linearised PLL.



$$\phi_e(t) = \phi(t) - 2\pi k_v \int_0^t v_L(\tau)d\tau$$

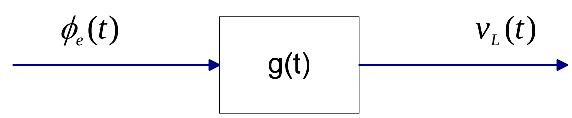
Equivalently, by differentiating we obtain:

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi(t)}{dt} - 2\pi k_v v_L(t)$$

Fourier transform of the above equation gives :

$$j2\pi f\phi_e(f) = j2\pi f\phi(f) - 2\pi k_v v_L(f)$$

Note:



$$v_L(f) = \phi_e(f).G(f)$$

$$FT[\frac{d}{dt}x(t)] = j2\pi f X(f)$$

$$\phi_e(f) = \frac{1}{1 + \frac{k_v}{jf}G(f)}\phi(f)$$

We design the loop filter such that $\left|\frac{k_v G(f)}{jf}\right| >> 1$ in the frequency band $|f| < W$.

$$\therefore \phi_e(f) = \frac{\phi(f)}{\frac{k_v}{jf}G(f)} = \frac{jf\phi(f)}{k_v G(f)}$$

$$\phi_e(f)G(f) = \frac{jf\phi(f)}{k_v} = \frac{jf\phi(f)}{k_v}$$

$$\underbrace{\phi_e(f)G(f)}_{v_L(f)}$$

$$v_L(f) = \frac{jf}{k_v}\phi(f)$$

$$v_L(f) = \frac{j2\pi f}{2\pi k_v}\phi(f)$$

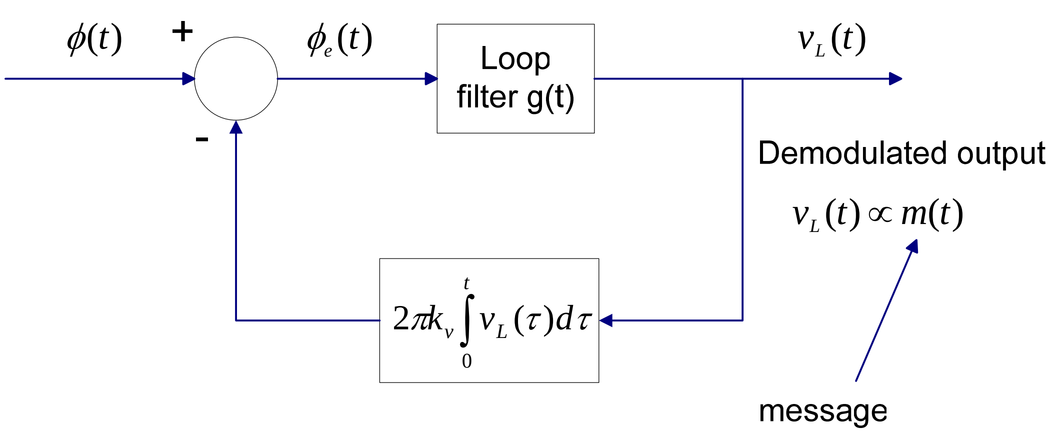
Take inverse FT

$$v_L(t) = \frac{1}{2\pi k_v}\frac{d}{dt}\left[2\pi k_f \int_0^t m(\tau)d\tau\right]$$

$$= \frac{1}{2\pi k_v} 2\pi k_f m(\tau)$$

$$\therefore v_L(t) = \frac{k_f}{k_v} m(t)$$

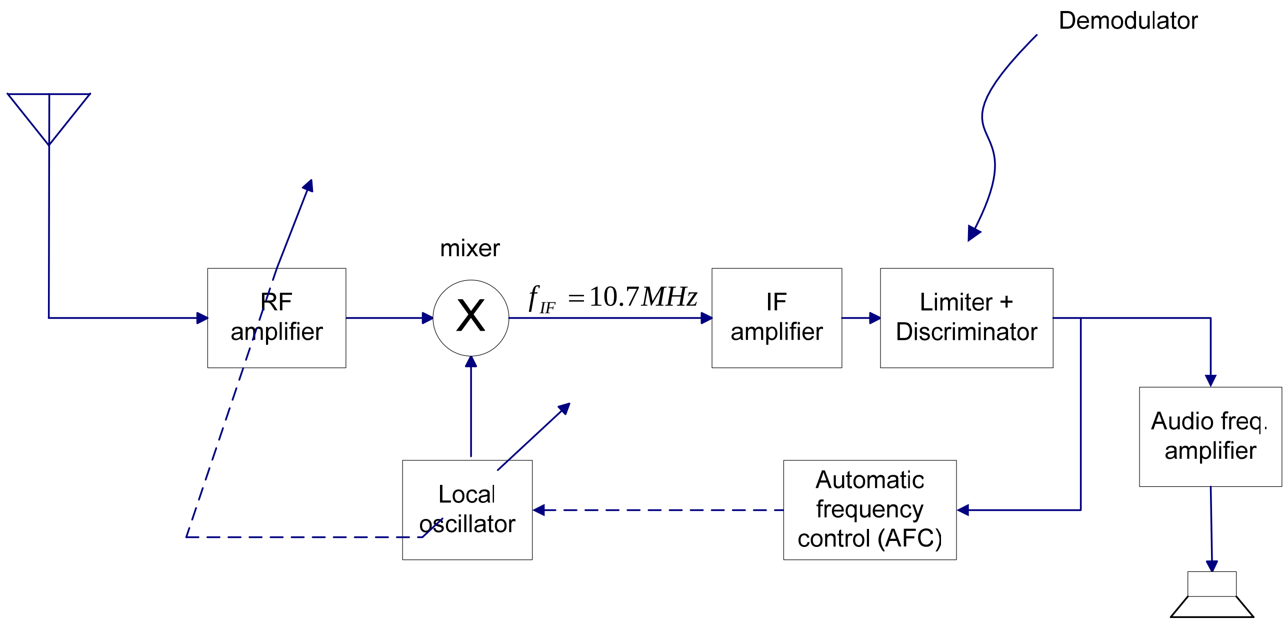
Since the control voltage of the VCD is proportional to the message signal $v_L(t)$ is the demodulated signal.



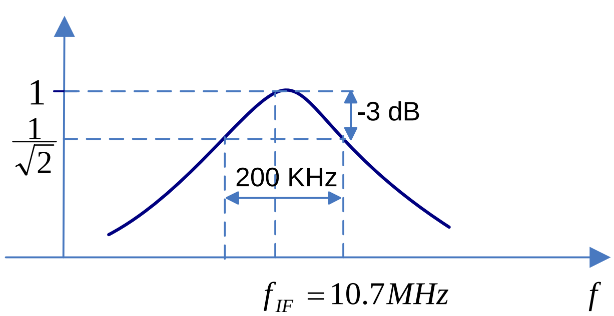
We observe the output of the loop filter with frequency responses G (f) is the desired message signal. Hence the bandwidth of G (f) should be the same as the bandwidth W of the message signal.

FM Radio Broadcasting (Mono transmission)

- Commercial FM radio broadcasting utilizes the frequency band 88-108MHz for transmission of voice and music signals.
- The carrier frequencies are separated by 200 KHz and the peak –frequency deviation is fixed at 75 KHz.
- The receiver most commonly used in FM radio broadcast is a super heterodyne type.



As in AM radio reception, common tuning between the RF amplification and the local oscillator allows the mixer to bring all FM signals to a common if bandwidth of 200 KHz, centered at $f_{IF} = 10.7MHz$.



Since the message signal m (t) is embedded in the frequency of the carrier, any amplitude variation in the received signal are a result of additive noise and interference. The amplitude limiter removes any amplitude variations in the received signal.

FM stereo Broadcasting

Many FM radio stations transmit music programs in stereo by using the outputs of two microphones placed in two different parts of the stage.
(see reference both for more details of the FM stereo transmitter.)