Fundamentals of Communications (XE37ZKT), Part I

Comparing the AM, FM, and PM

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4th

1 Outline

- Angle modulations: FM
    - Integrating the phase
    - Sine and cosine cases
- Angle modulations: PM
    - Modulating the phase
    - Sine and cosine cases
- Noise properties: FM
    - SNR enhancements after the demodulation
    - Threshold levels
- Noise properties: AM
    - SNR enhancements after the demodulation
    - Threshold levels

2 Angle Modulations: FM

A general formula defining the internal angle for the FM (frequency modulation, ν_m is a modulating signal):

$$\frac{d\varphi(t)}{dt} = \nu_m(t) \Rightarrow \varphi(t) = \int_0^t \nu_m(t')\,dt'$$

Solving for the cases of sinusoidal or cosinusoidal modulating signals:

Sine: $\displaystyle\int_0^t \sin(\omega_m t')\,dt' = \frac{1}{\omega_m}\left[-\cos(\omega_m t')\right]_0^t$

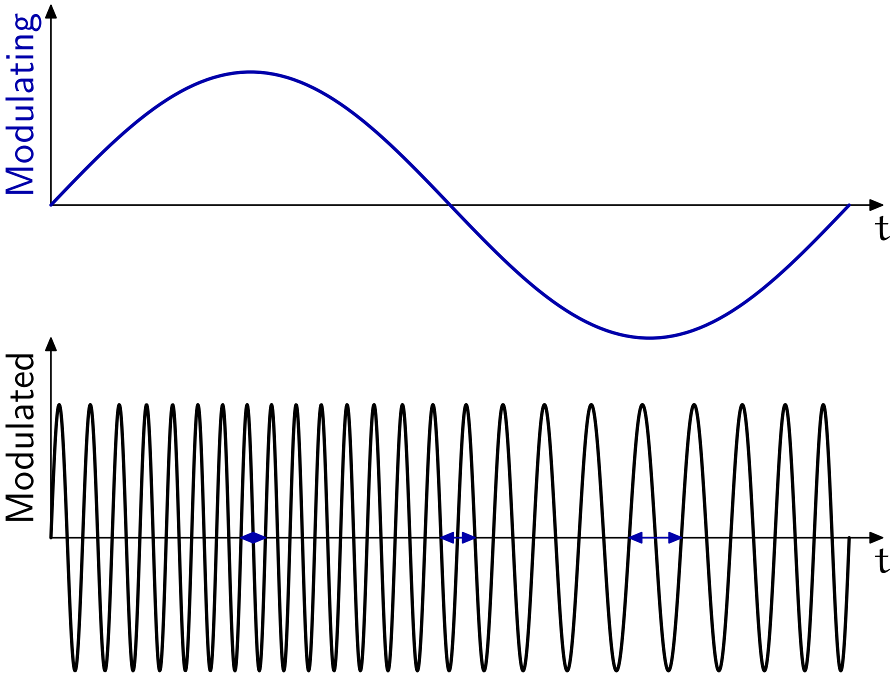
$$= \frac{1}{\omega_m}\left[1 - \cos(\omega_m t)\right] \Rightarrow \sin\left[\omega_c t + b\left(1 - \cos(\omega_m t)\right)\right]$$

Cosine: $\displaystyle\int_0^t \cos(\omega_m t')\,dt' = \frac{1}{\omega_m}\left[\sin(\omega_m t')\right]_0^t$

$$= \frac{1}{\omega_m}\sin(\omega_m t) \Rightarrow \cos\left[\omega_c t + b\sin(\omega_m t)\right]$$

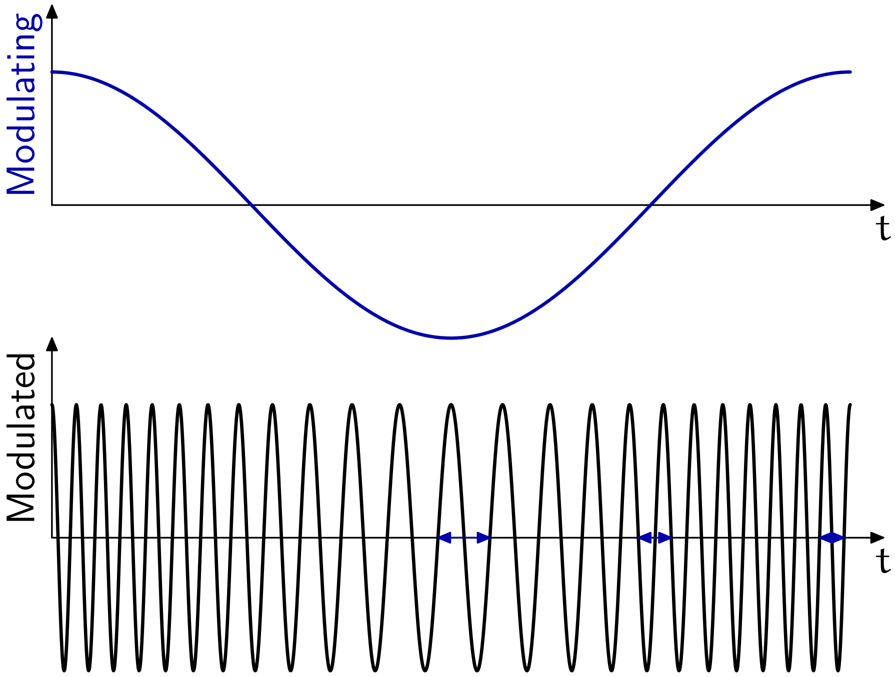
2.1 **FM:** Carrier Sine, Modulating Sine

$$\sin\left[\omega_c t + b\big(1 - \cos(\omega_m t)\big)\right],\ \frac{\omega_c}{\omega_m} = 24,\ b = 500$$



2.2 FM: Carrier Cosine, Modulating Cosine

$$\cos\left[\omega_c t + b\sin(\omega_m t)\right],\ \frac{\omega_c}{\omega_m} = 24,\ b = 500$$



3 Angle Modulations: PM

As the opposite of FM, the phase is proportional to the modulating signal (not to its integral as that in FM), i.e.

$$\varphi(t) = v_m(t)$$

For the continuity requirements, the two following cases are possible:

Sine: $v_m(t) = \sin(\omega t)$

$\Rightarrow \sin\left[\omega_c t + \beta \sin(\omega_m t)\right]$

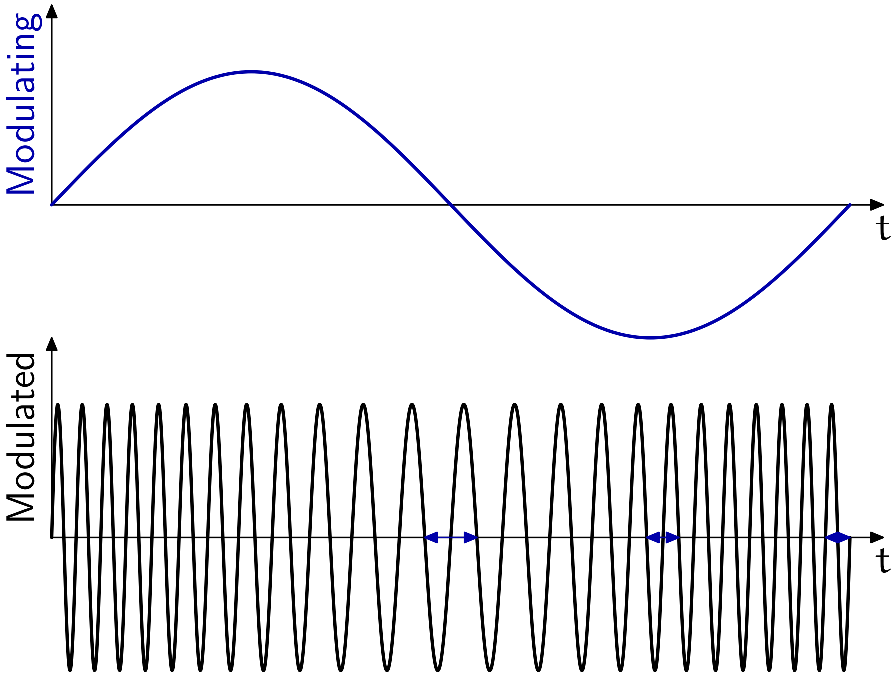
Cosine: $v_m(t) = \cos(\omega t)$

$\Rightarrow \cos\left[\omega_c t - \beta\left(1 - \cos(\omega_m t)\right)\right]$

As a general result, the instantaneous carrier frequency is proportional to the *derivative* of the modulating signal (not to its magnitude as that in FM).

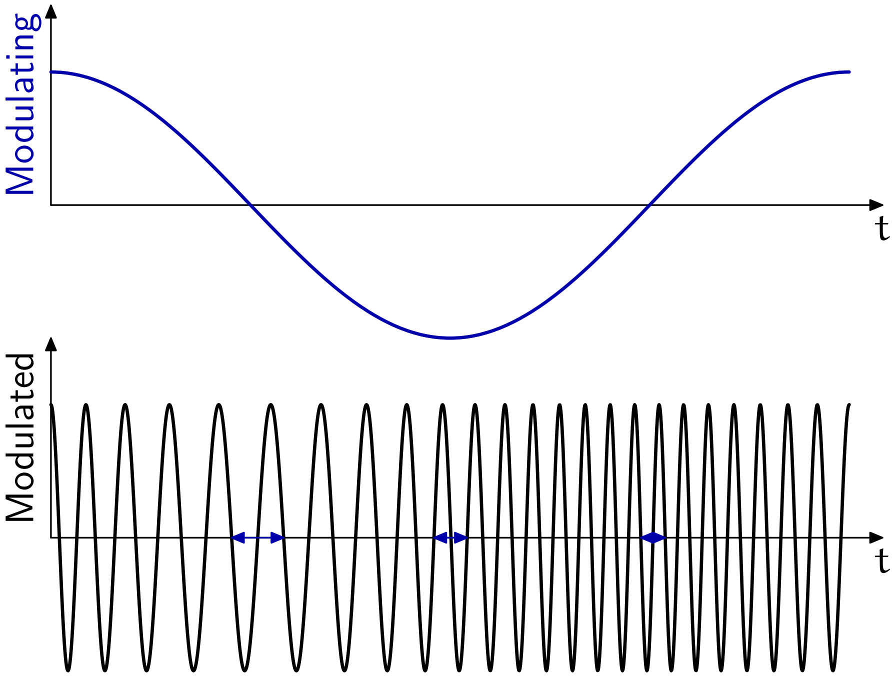
3.1 **PM:** Carrier Sine, Modulating Sine

$$\sin\left[\omega_c t + \beta \sin(\omega_m t)\right],\ \frac{\omega_c}{\omega_m} = 24,\ \beta = 500$$



3.2 **PM:** Carrier Cosine, Modulating Cosine

$$\cos\left[\omega_c t - \beta\left(1 - \cos(\omega_m t)\right)\right],\ \frac{\omega_c}{\omega_m} = 24,\ \beta = 500$$



4 Noise Properties: FM

An enhancement of the signal-noise-ratio (SNR) after the FM demodulation:

$$\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C} = \begin{cases} \frac{3}{2}\beta^2 & \text{without deemphasis,} \\ \frac{1}{2}\left(\frac{f_m}{f_{de}}\right)^2 \beta^2 & \text{with deemphasis,} \end{cases}$$

where f_{de} is the deemphasis cutoff frequency. For the standard FM parameters ($\beta = 5$, $f_m = 15$ kHz, $f_{de} = 2.1$ kHz), the enhancements are the following:

$$\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C} = \begin{cases} 15.7 \text{ dB} & \text{without deemphasis,} \\ 28 \text{ dB} & \text{with deemphasis.} \end{cases}$$

For $\beta = 2$ and the same f_m and f_{de}, the enhancements are worse (see the comparison):

$$\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C} = \begin{cases} 7.8 \text{ dB} & \text{without deemphasis,} \\ 20 \text{ dB} & \text{with deemphasis.} \end{cases}$$

The noise threshold of the FM modulation can be estimated by the formula

$$20(\beta + 2),$$

which gives the cutoff levels 21.5 dB and 16 dB for $\beta = 5$ and $\beta = 2$, respectively (see the comparison).

Entire expression for the signal-noise-ratio after the demodulation can be found in the Carlson's text book:[1]

$$(\mathsf{SNR})_O = \frac{\frac{3}{2}\beta^2 (\mathsf{SNR})_C}{1 + \frac{12\beta}{\pi} (\mathsf{SNR})_C \exp\left(-\frac{(\mathsf{SNR})_C}{2(\beta+2)}\right)}$$

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[1] A. B. Carlson, Communication Systems, McGraw-Hill 1975.

5 Noise Properties: AM

An enhancement of the signal-noise-ratio (SNR) after the AM demodulation:

$$\frac{(\mathrm{SNR})_O}{(\mathrm{SNR})_C} = \frac{m^2}{m^2+2},$$

which gives the values -4.8 dB and -13.7 dB for the modulation depths $m = 1$ and $m = 0.3$, respectively (see the comparison).
The level threshold for the 100 % modulation is approximated by 13 dB (see the comparison).
Entire expression for the signal-noise-ratio after the demodulation can again be found in the Carlson's text book:

$$(\mathrm{SNR})_O = \frac{\frac{m^2}{m^2+2}(\mathrm{SNR})_C}{1+\exp\left(-\frac{(\mathrm{SNR})_C}{4}\right)}$$

Comparison of the FM, DSB, and AM Noise Properties